Total No. of Questions—**12**] [Total No. of Printed Pages—**7**

Seat No.	

[4956]-1

F.E. EXAMINATION, 2016
ENGINEERING MATHEMATICS-I
(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and three questions from Section II.

(*ii*) Answer to the two Sections should be written in separate asnswer-book.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Reduce the matrix :

$$A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$$

to echelon form and determine its rank. [5]

(*b*) Discuss consistency and solve if consistent : [6]

$$\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8 \\ 2x - 2y + 3z &= 7 \end{aligned}$$

(*c*) Verify Cayley-Hamilton theorem for the matrix : [6]

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Or

2. (*a*) Find eigen values and eigen vectors for the matrix : [6]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(*b*) Examine for linear dependence or linear independence of vectors :

$x_1 = (1, 1, -1)$, $x_2 = (2, 3, -5)$, $x_3 = (2, -1, 4)$

If dependent, find the relation between them. [6]

(*c*) Show that :

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. Find A^{-1}. [5]

3. (*a*) Find z if $\arg(z + 2i) = \frac{\pi}{4}$ and $\arg(z - 2i) = \frac{3\pi}{4}$. [5]

(*b*) Find all values of $(1 + i)^{1/5}$, show that their product is $1 + i$. [5]

(*c*) If $\log(\tan x) = y$ prove that : [6]

(*i*) $\cosh ny = \frac{1}{2}(\tan^n x + \cot^n x)$

(*ii*) $\sinh ny = \frac{1}{2}(\tan^n x - \cot^n x)$

Or

4. (*a*) If

$$2\cos\phi = x + \frac{1}{x} \text{ and } 2\cos\theta = y + \frac{1}{y}$$

then prove that : [5]

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\phi + n\theta)$$

(*b*) Prove that i^i is wholly real and find its principal value. Also show that the values of i^i form a G.P. [5]

(*c*) If $\tan(\alpha + i\beta) = x + iy$ prove that :

(*i*) $x^2 + y^2 + 2x \cot 2\alpha = 1$

(*ii*) $x^2 + y^2 - 2y \coth 2\beta + 1 = 0$ [6]

5. (*a*) Find *n*th derivative of : [5]

$$\frac{1}{x^2 - 4x + 3}$$

(*b*) If

$$y = \mathrm{A}\cos\log x + \mathrm{B}\sin\log x$$

then prove that :

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$ [6]

(*c*) Discuss convergence or divergence (any *one*) : [6]

(*i*) $\sum_{n=1}^{\infty} \frac{2^n + 1}{3^n + 1}$

(*ii*) $\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \ldots\ldots. + \ldots$

Or

6. (*a*) Find *n*th derivative of : [5]

$$y = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$$

(*b*) If

$$y = (x + \sqrt{x^2 - 1})^m$$

then prove that : [6]

$$(x^2 - 1)y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2)\, y_n = 0$$

(*c*) Attempt (any *one*) : [6]

(*i*) Test absolute or conditional convergence of :

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \ldots\ldots$$

(*ii*) Determine the interval of convergence for the series :

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

SECTION II

7. (*a*) Expand $(1 + x)^x$ in a series upto a term in x^4. [6]

(*b*) Expand :

$$x^4 - 3x^3 + 2x^2 - x + 1$$

in power of $(x - 3)$. [5]

(*c*) Attempt the following (any *one*) : [6]

(*i*) Evaluate :

$$\lim_{x \to 0} \frac{e^{2x} - (1 + x)^2}{x \log (1 + x)}$$

(*ii*) If

$$\lim_{x \to 0} \frac{\sin 2x + p \sin x}{x^3}$$

is finite then find the value of p and hence the value of the limit.

Or

8. (*a*) Expand :

$$\log \cos \left(x + \frac{\pi}{4} \right)$$

using Taylor's theorem in ascending powers of x. [6]

(*b*) Expand $\sqrt{1 + \sin x}$ up to x^6. [5]

(*c*) Attempt the following (any *one*) : [6]

(*i*) Evaluate :

$$\lim_{x \to \infty} \left\{ x - x^2 \log \left(1 + \frac{1}{x} \right) \right\}$$

(*ii*) Find a, b, c if :

$$\lim_{x \to \infty} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$$

9. (*a*) If $u = x^y$, then show that : [5]

$$\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial z}$$

(*b*) If $x^2 = au + bv$, $y = au - bv$, then prove that : [6]

$$\left(\frac{\partial u}{\partial x} \right)_y \left(\frac{\partial x}{\partial u} \right)_v = \left(\frac{\partial v}{\partial y} \right)_x \left(\frac{\partial y}{\partial v} \right)_u$$

(*c*) If [6]

$$u = \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos\left(\frac{xy + yz}{x^2 + y^2 + z^2}\right)$$

show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 4\left(\frac{x^2 y^2 z^2}{x^2 + y^2 + z^2}\right)$$

Or

10. (*a*) If $u = f(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$, then prove that : [6]

$$u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r} \mathrm{F}'(r)$$

(*b*) If

$$v = f(e^{x-y}, e^{y-z}, e^{z-x})$$

then show that : [5]

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0.$$

(*c*) If

$$u = \operatorname{cosec}^{-1} \sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$$

show that : [6]

$$x^2 \frac{2\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

11. (*a*) For the transformation : [5]

$$x = e^u \cos v, \; y = e^u \sin v$$

prove that JJ' = 1.

(*b*) If [5]

$$u^2 + xv^2 - uxy = 0, \; v^2 - xy^2 + 2uv + u^2 = 0$$

find $\frac{\partial u}{\partial x}$ by proper choice of dependent and independent variable. [5]

(*c*) Find maximum value of : [6]

$$x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$$

Or

12. (*a*) If

$$u = x + y + z, v = x^2 + y^2 + z^2, \; w = xy + yz + zx$$

examine whether u, v, w are functionally dependent. If so, find the relation between them. [5]

(*b*) Find the possible percentage error in computing the parallel resistance 'r' of three resistance r_1, r_2, r_3 from the formula :

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

if r_1, r_2, r_3 are each in error by 1.2 %. [5]

(*c*) Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum. [6]